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Ohmic heating of complex fluids

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Abstract—This paper describes the basic principles and physical modelling of systems based on the ohmic heating of liquids by passage of mains frequency electric current through the liquid itself, in order to heat it on a continuous flow regime. Preliminary numerical results for non-Newtonian fluids of Herschel–Bulckley type (Carbopol 940) are presented and the effect of natural convection, and shear and temperature dependence are emphasized.

1. INTRODUCTION

IN MANY industrial processes one is confronted with heating of very viscous poor conducting fluids or fluids with complex thermorheological behaviours such as non-Newtonian fluids. Non-Newtonian fluids are those which have a non-linear relationship between shear stress τ and shear rate γ. These fluids are widely used in daily life. Plastic is one of the products that in a melt state shows a strong non-Newtonian behaviour. It is reported [1] that the annual consumption of plastic in the United States of America has increased from 4.5 to 20 billion kilograms in a period of 16 years between 1962 and 1979. Polymer solutions are another type of non-Newtonian fluids. They are present in toothpaste, cosmetic make-up, aerosols, paints, automobile polishes and different pharmaceutical and household products. They are also used as thickening, suspending and stabilizing agents for food products. The worldwide use of non-Newtonian fluids is constantly increasing, therefore it is important to understand their transport behaviour, especially with regard to their heating and cooling. They are, in general, highly thermodependent and their behaviour varies drastically with the shear stress imposed on them by their environment. Therefore novel heating techniques which require less agitation of the fluid and also avoid local extra heating are strongly desired in non-Newtonian fluid processing. Most heating techniques used in industrial applications, however, are indirect, in which heat is applied to a contact surface, then it is transferred to the fluid by conduction, convection or radiation mechanisms. Other alternatives such as heating by a condensing steam are also common practice. Nevertheless, these techniques quite often fail to satisfy certain operational requirements such as preservation of the quality of the fluid or avoiding overheating and burning.

Overheating of poor conducting liquids adjacent to hot surfaces cannot only cause burning or degradation of the liquids, but it is at the origin of heat exchanger fouling. Furthermore, in the classical hot surface heating method, the heating capacity of the system is restricted by the available surface area of the heat exchanger. Condensing steam heating used in sterilizing processes also has the disadvantage of additive water which has to be removed after the process. The additive water often changes the taste of the food product after sterilization (for example, in the case of chocolate) and water removal consumes extra energy, and hence is a more costly process.

An alternative method to obviate the basic difficulties of heating by heat conduction from a hot surface heat exchanger is the direct resistance or ohmic heating [2, 3]. It consists of causing an electric current to flow directly in the flowing media between a pair of electrodes. This technique can be applied to liquids containing free ions, and therefore which are, electric conductors. Different ions present in the liquid start to move once the liquid is under the influence of an alternating electric field. This permanent movement of ions is accompanied by a heat release in the liquid bulk that can be formulated by the Joule law. In this manner, direct resistance bulk heating offers a major advantage in comparison to the conventional heating methods for viscous, poor thermal conducting liquids or particulates. It should be emphasized here that we are not concerned with increasing, as is generally the case, the heat transfer coefficient between the hot wall and the fluid flowing on it. Here, we are interested in a totally different technique, which consists of heating every fluid element in the place where it is, therefore the heat transfer between the hot wall and the fluid is irrelevant as there is no hot wall at all. This is the basic difference between an ohmic heating device, as we describe it here, and all other devices classified under 'heat exchangers', including those which electrically heat the fluids. *In situ* and laboratory exper-

NOMENCLATURE

A	channel aspect ratio (length to width ratio)
a	mean thermal diffusivity of fluid [$\mathrm{m^2\,s^{-1}}$]
c_p	constant pressure specific heat [$\mathrm{J\,kg^{-1}\,K^{-1}}$]
$\mathrm{D}/\mathrm{D}t$	total derivative
$\vec{E}$	electric field [$\mathrm{V\,m^{-1}}$]
$\vec{f}_e$	electric force [N]
g	gravitation acceleration [$\mathrm{m\,s^{-2}}$]
Gr	Grashof number
H	electrode length [m]
I	current intensity [A]
l	inter-electrode distance [m]
p	pressure [Pa]
Pe	Peclet number
Pr	Prandtl number
Re	Reynolds number
Ri	Richardson number
t	time [s]
T	temperature [K]
T_e	temperature at the entrance to the channel [K]
T_s	temperature at the channel exit section [K]
T_0	reference mean temperature [K]
u	spanwise velocity component [$\mathrm{m\,s^{-1}}$]
U	electric potential [V]
U_0	potential difference [V]
v	streamwise velocity component [$\mathrm{m\,s^{-1}}$]
$\vec{\mathrm{V}}$	velocity vector [$\mathrm{m\,s^{-1}}$]
V_m	mean streamwise velocity [$\mathrm{m\,s^{-1}}$].
Greek symbols	
β	expansion coefficient of fluid at constant pressure
$\dot{\gamma}$	strain rate [$\mathrm{s^{-1}}$]
Δ	Laplacian operator
ε	electric permitivity [$\mathrm{C\,V^{-1}\,m^{-1}}$]
λ	thermal conductivity [$\mathrm{W\,m^{-1}\,K^{-1}}$]
μ	viscosity [$\mathrm{kg\,m^{-1}\,s^{-1}}$]
ν	kinematic viscosity [$\mathrm{m^2\,s^{-1}}$]
ρ	specific mass [$\mathrm{kg\,m^{-3}}$]
ρ_e	free charge density [$\mathrm{C\,m^{-3}}$]
σ	electric conductivity [$\mathrm{S\,m^{-1}}$]
τ	viscous stress tensor
$\vec{\nabla}$	gradient
$\nabla.$	divergence
∇_x	curl.

iments have proved the efficiency of the ohmic heating method. A review of the industrial applications of ohmic heating can be found in ref. [4].

From the fundamental point of view, direct resistance heating of liquids is a complex physical problem in which a strong interaction of heat transfer, hydrodynamic and electric phenomena can be observed [4]. In fact the thermophysical properties of the liquid in flow are functions of temperature, and the temperature field in its turn depends on the residence time in the system (in which the fluid is under the effect of the electric field), i.e. the flow field. In a realistic configuration in which there exists a velocity difference between the fluid flowing in the core and the fluid flowing close to the wall (boundary layer), the resultant temperature gradient can generate a free convection flow superimposed on the main stream. The effect of natural convection can appear as an accelerated flow field adjacent to the electrodes for a vertical channel. Interdependence between the thermal, hydrodynamic and electric fields is completed by the temperature dependence of electric conductivity of the working fluid. A realistic analysis of the problem should take into account this interdependence and its consequences.

To analyse the problem of ohmic heating of liquids we have developed a physical model for formulation of the problem and derivation of the equations which govern the phenomenon. The equations are then numerically solved. An experimental apparatus is also specifically designed and constructed for validation of the numerical results. Its description has been reported elsewhere [3].

2. THE PHYSICAL MODEL

The equations governing the conservation of mass, momentum, energy and the electrostatic equations for an incompressible fluid can be written in their general form as follows [5]:

Equation of conservation of mass

$$\nabla\cdot\vec{\mathrm{V}} = 0. \qquad (1)$$

Equation of the conservation of momentum

$$\rho\frac{\mathrm{D}\vec{\mathrm{V}}}{\mathrm{D}t} = -\vec{\nabla}p - \rho\vec{g} + \vec{f}_e + \nabla\cdot\tau \qquad (2)$$

where

$$\tau = \mu\left(\frac{\partial Vi}{\partial xj} + \frac{\partial Vj}{\partial xi}\right)$$

and f_e is a body force due to the electric field. The electric body force is composed of different parts and can be written as

$$\vec{f}_e = \rho_e\vec{E} - \frac{1}{2}E^2\vec{\nabla}\varepsilon - \vec{\nabla}\left(\frac{1}{2}\rho E^2\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right). \qquad (3)$$

In the expression of the electric force the term $\rho_e\vec{E}$ represents the force per volume on the fluid due to the action of the electric field on the free charges of density ρ_e. This term is the dominant part of the electrohydrodynamic force in the presence of a continuous electric field acting on a conducting fluid or solid.

The term $-\frac{1}{2}E^2\vec{\nabla}\varepsilon$ is due to the action of the electric field on electric dipoles (forces of polarization) when the electric permitivity is not uniform. This term is generally negligible in comparison with the previous term and becomes considerable only if an alternative electric field acts on a dielectric liquid.

The term $-\vec{\nabla}(\frac{1}{2}\rho E^2(\partial\varepsilon/\partial\rho)_T)$ represents the effect of electrostriction, and can be absorbed in the pressure term of the momentum equation. For the case of an incompressible fluid this term is negligible.

From equation (2) we note that the hydrodynamics of the problem can be influenced by the electric force $\vec{f}_e$ and the temperature field. In fact, the temperature dependence of the thermophysical properties of the liquid can cause bouyancy effects superimposed on the main flow and, as we will observe later, cause a mixed convection phenomenon.

Equation of the conservation of energy

Taking into consideration the viscous and electric dissipation, one can write the equation of the conservation of energy as

$$\rho c_p \frac{\mathrm{D}T}{\mathrm{D}t} = \nabla\cdot(\lambda\vec{\nabla}T)+\sigma E^2+\tau:\bar{\bar{\nabla}}V. \tag{4}$$

We observe that the coupling between heat transfer and the hydrodynamic of the problem is through the nonlinear transport term and the viscous dissipation term while the coupling between electric and heat transfer is secured by the electric dissipation source term.

Electric equation

The electrostatic equations for a linear ohmic dielectric are

$$\nabla\cdot(\varepsilon\vec{E}) = \rho_e \tag{5}$$

$$\nabla x\vec{E} = 0 \tag{6}$$

$$\nabla\cdot(\vec{\mathrm{J}})+\frac{\partial\rho_e}{\partial t} = 0 \tag{7}$$

where

$$\vec{\mathrm{J}} = \rho_e\vec{\mathrm{V}}+\sigma\vec{E} \quad \text{and} \quad \vec{E} = -\vec{\nabla}U. \tag{8}$$

We restrict ourselves here to a rectangular cell in which two opposite walls constitute the electrodes, and the electrode width is large compared with the distance separating them. We consider an ascending flow between two vertical parallel plates. In this case the pertinent spatial variables are the abscissa x perpendicular to the electrodes and the ordinate y pointing in the flow direction, and the momentum equation resulting from equation (1) can be written as

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)+f_{ex} \tag{9}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y}-\rho g+\frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial x}\left(\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right)+f_{ey} \tag{10}$$

or in a more explicit form

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial\mu}{\partial x}\frac{\partial u}{\partial x}+\frac{\partial\mu}{\partial y}\frac{\partial v}{\partial x}+\left(\mu\frac{\partial^2 u}{\partial x^2}\right)+\left(\mu\frac{\partial^2 v}{\partial x\partial y}\right)+f_{ex} \tag{11}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y}-\rho g+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial\mu}{\partial y}\frac{\partial v}{\partial y}+\frac{\partial\mu}{\partial x}\frac{\partial u}{\partial y}+\left(\mu\frac{\partial^2 v}{\partial y^2}\right)+\left(\mu\frac{\partial^2 u}{\partial x\partial y}\right)+f_{ey}. \tag{12}$$

In the case of an ionic liquid, we keep only the term $\rho_e\vec{E}$ of the electric force $\vec{f}_e$ given by equation (3). Below we shall determine the expression for this electric force. Equation (5) can be written as

$$\nabla\cdot\vec{E}+\frac{\partial\varepsilon}{\varepsilon\partial T}\vec{E}\cdot\vec{\nabla}T = \frac{\rho_e}{\varepsilon} \tag{13}$$

and equations (7) and (8) give

$$\nabla\cdot\vec{E}+\frac{\partial\sigma}{\sigma\,\partial T}\vec{E}\cdot\vec{\nabla}T+\frac{\mathrm{D}}{\sigma\mathrm{D}t}(\rho_e) = 0. \tag{14}$$

By substracting equation (14) from equation (13) we obtain

$$\frac{\mathrm{D}}{\sigma\mathrm{D}t}(\rho_e)+\frac{\rho_e}{\varepsilon} = \left(\frac{\partial\varepsilon}{\varepsilon\,\partial T}-\frac{\partial\sigma}{\sigma\,\partial T}\right)\vec{E}\cdot\vec{\nabla}T. \tag{15}$$

Using the relation $\tau_\sigma = \varepsilon/\sigma$ and by multiplying the equation (15) by ε, we have

$$\tau_\sigma\frac{\mathrm{D}}{\mathrm{D}t}(\rho_e)+\rho_e = \left(\frac{\partial\varepsilon}{\partial T}-\tau_\sigma\frac{\partial\sigma}{\partial T}\right)\vec{E}\cdot\vec{\nabla}T. \tag{16}$$

Equation (16) is in fact a differential equation describing the variation of free charge density ρ_e of a fluid element moving with velocity $\vec{\mathrm{V}}$. This equation in ρ_e yields the solution

$$\rho_e = \left(\frac{\partial\varepsilon}{\partial T} - \tau_\sigma \frac{\partial\sigma}{\partial T}\right)\vec{E}\cdot\vec{\nabla}T + \mathrm{C}\,\mathrm{e}^{-t/\tau_\sigma} \tag{17}$$

in which C is a constant.

In general, and particularly in our case, the characteristic time for free charge relaxation is very small ($\tau_\sigma \equiv 3\cdot 10^{-8}$), in such a way that we have

$$\rho_e = \left(\frac{\partial\varepsilon}{\partial T} - \tau_\sigma \frac{\partial\sigma}{\partial T}\right)\vec{E}\cdot\vec{\nabla}T. \tag{18}$$

Then the electric force becomes

$$\vec{f}_e = \rho_e\vec{E} = \left(\frac{\partial\varepsilon}{\partial T} - \tau_\sigma \frac{\partial\sigma}{\partial T}\right)(\vec{E}\vec{\nabla}T)\vec{E}. \tag{19}$$

It should be noted that this force vector is colinear with $\vec{E}$ and therefore has two components, f_{ex} and f_{ey}.

$$f_{ex} = \left(\frac{\partial\varepsilon}{\partial T} - \tau_\sigma \frac{\partial\sigma}{\partial T}\right)\left(E_x\frac{\partial T}{\partial x} + E_y\frac{\partial T}{\partial y}\right)E_x \tag{20}$$

is perpendicular to the electrode surface and its effect is to bring the fluid elements towards it, and

$$f_{ey} = \left(\frac{\partial\varepsilon}{\partial T} - \tau_\sigma \frac{\partial\sigma}{\partial T}\right)\left(E_x\frac{\partial T}{\partial x} + E_y\frac{\partial T}{\partial y}\right)E_y \tag{21}$$

is parallel to the electrode.

By taking into consideration the equation of the conservation of mass (1), and assuming a weak variation of viscosity, the coupled system of equations becomes

$$\nabla\cdot\mathbf{V} = 0$$

$$\rho\frac{\mathrm{D}\vec{\mathbf{V}}}{\mathrm{d}t} = -\vec{\nabla}P - \rho\vec{g} + \nabla\cdot(\mu(T)\bar{\bar{\nabla}}V) + \vec{f}_e$$

$$\rho c_p\frac{\mathrm{D}T}{\mathrm{D}t} = \nabla\cdot(\lambda(T)\vec{\nabla}T) + \sigma(T)(\vec{\nabla}U)^2 + \Phi$$

$$\nabla\cdot(-\sigma(T)\vec{\nabla}U) = 0. \tag{22}$$

This system of equations should be solved in conjunction with the hydrodynamic, thermal and electric boundary conditions. They can be in the form of prescribed velocity, temperature and electric potential, or thermally and electrically isolated boundaries.

Non-dimensional equations

For non-dimensionalizing the equations, we use interelectrode distance l and mean velocity V_m as characteristic length and velocity, respectively. In order to treat the free convection problem we use the Boussinesq approximation, i.e. the fluid density is constant and equal to ρ_0 everywhere except in the bouyancy force term. ρ_0 is the fluid density at the mean temperature T_0 defined as

$$T_0 = \frac{(T_s + T_e)}{2}. \tag{23}$$

In the equation of motion, the variable p is in fact the hydrostatic pressure difference. The pressure P is then given by

$$P = p - \rho_0 g y. \tag{24}$$

We have, therefore,

$$-\frac{\partial P}{\partial y} - \rho g = g(\rho_0 - \rho) - \frac{\partial p}{\partial y}. \tag{25}$$

The coefficient of thermal dilatation at constant pressure β is given by

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \tag{26}$$

or

$$\beta = \frac{1}{V}\left(\frac{V - V_0}{T - T_0}\right). \tag{27}$$

It can also be expressed in terms of density as

$$\beta = \frac{\rho_0 - \rho}{\rho_0(T - T_0)} \tag{28}$$

or

$$\rho_0 - \rho = \beta\rho_0(T - T_0). \tag{29}$$

This gives rise to a linear variation of the density with temperature

$$\rho = \rho_0[1 - \beta(T - T_0)]. \tag{30}$$

The relations (25) and (30) give

$$-\frac{\partial P}{\partial y} - \rho g = \beta g(T - T_0) - \frac{\partial p}{\partial y}. \tag{31}$$

By developing the equations of motion, we have

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \left(\frac{\partial\varepsilon}{\partial T} - \tau_\sigma\frac{\partial\sigma}{\partial T}\right)E^2\frac{\partial T}{\partial x} \tag{32}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) + \beta\rho_0 g(T - T_0) + \left(\frac{\partial\varepsilon}{\partial T} - \tau_\sigma\frac{\partial\sigma}{\partial T}\right)E^2\frac{\partial T}{\partial y}. \tag{33}$$

By choosing the other thermophysical properties of the fluid at the reference temperature $T_0 = (T_s + T_e)/2$ and denoting them as $\lambda_0 = \lambda(T_0)$, $\mu_0 = \mu(T_0)$, $\sigma_0 = \sigma(T_0)$, $\varepsilon_0 = \varepsilon(T_0)$, and $\rho_0 = \rho(T_0)$, we can define the dimensionless variables and parameters as

$$x^* = \frac{x}{l};\quad y^* = \frac{y}{l};\quad t^* = \frac{tV_m}{l};$$

$$u^* = \frac{u}{V_m};\quad v^* = \frac{v}{V_m};\quad p^* = \frac{p}{\rho_0 v_m^2};$$

$$T^* = \frac{(T - T_0)}{(T_s - T_e)};\quad U^* = \frac{U}{U_0};\quad \sigma^* = \frac{\sigma(T)}{\sigma_0};$$

$$\varepsilon^* = \frac{\varepsilon(T)}{\varepsilon_0}; \quad \lambda^* = \frac{\lambda(T)}{\lambda_0} \tag{34}$$

$$\mu^* = \frac{\mu(T)}{\mu_0}; \quad \Phi^* = \frac{\Phi l}{\rho_0 c_p \Delta T V_m}, \quad Re = \frac{V_m l}{v_0};$$

$$Pr = \frac{v_0}{a_0}; \quad Pe = Pr \cdot Re; \quad Gr = \frac{\beta g l^3 (T_s - T_e)}{v_0^2};$$

$$Ri = \frac{Gr}{Re^2} = \frac{\beta g U I}{\rho_0 c_p V_m^3}; \quad A = \frac{H}{l};$$

$$Gr_{el} = \left(\frac{\partial \varepsilon}{\partial T} - \tau_\sigma \frac{\partial \sigma}{\partial T}\right) \frac{\rho_0 E^2 \Delta T l^2}{\mu_0^2}; \quad Ri_{el} = \frac{Gr_{el}}{Re^2}.$$

The complete system of equations, i.e. equations of motion, energy and electrostatic can then be written in the non-dimensional form as

$$\nabla \cdot \vec{V}^* = 0$$

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x} + \frac{1}{Re}\left(\frac{\partial}{\partial x^*}\left(\mu^* \frac{\partial u^*}{\partial x^*}\right) + \frac{\partial}{\partial y^*}\left(\mu^* \frac{\partial u^*}{\partial y^*}\right)\right) + Ri_{el}\left(\frac{\partial T^*}{\partial x^*}\right)$$

$$\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial y} + \frac{1}{Re}\left(\frac{\partial}{\partial x^*}\left(\mu^* \frac{\partial v^*}{\partial x^*}\right) + \frac{\partial}{\partial y^*}\left(\mu^* \frac{\partial v^*}{\partial y^*}\right)\right) + Ri \cdot T^* + Ri_{el}\left(\frac{\partial T^*}{\partial y^*}\right)$$

$$\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Pe}\nabla \cdot (\lambda^* \nabla T^*) + \frac{1}{A}\sigma^* (\nabla U^*)^2 + \Phi^*$$

$$\nabla \cdot (-\sigma^* \vec{\nabla} U^*) = 0. \tag{35}$$

It is governed by five non-dimensional numbers (A, Re, Pe, Ri and Ri_{el}). Each represents the following characteristics of the system:

—The aspect ratio, A, characterizes the geometry of the system.

—The Reynolds number, Re, represents the ratio of inertial effects to the viscous effects.

—The Peclet number, Pe, characterizes the ratio of heat transfer by convection to heat transfer by conduction.

—The Richardson number, Ri, allows to evaluate the relative importance of the free convection due to the dissipation of electric energy to the forced convection. In fact Ri depends on the dissipated electric energy. Therefore this number reflects the coupling between electric and thermal phenomena.

—Ri_{el} is a non-dimensional number analogous to the Richardson number. It is constructed from the ratio of two electric non-dimensional numbers, Gr_{el} equivalent to the Grashof number and the square of the Reynolds number. Gr_{el} characterizes the electroconvection regime.

Given the complexity of the problem arising from the coupling between the different physical phenomena and due to the nonlinearity of the equations, we have restricted this investigation to a simple geometry which correspond also to some practical applications, and we have made some simplifying assumptions.

Hypotheses and simplified equations

—As the first simplification, we have fixed the geometry to a channel flow preceded by a diffuser at the entrance and followed by a nozzle at the exit. The larger sides of the channel constitute the electrodes.

—The working fluid is a Newtonian or non-Newtonian, incompressible ionic fluid for which we assume that the three terms of electric force are negligible. This assumption is justified by the fact that the electric field is alternating, therefore it is almost uniform in the volume and the volume density of the electric charge is null.

—The flow speed is small, thus the viscous dissipation is negligible.

—The thermophysical properties of the fluid (λ, μ, σ) depend on the temperature T, except for the non-Newtonian fluids where λ and μ depend also on the shear rate according to the constitutive laws which shall be prescribed further.

—The temperatue at the channel entrance T_e is uniform.

After the above assumptions, the system of equations to solve, in their compact form, reduce to

$$\nabla \cdot \vec{V} = 0$$

$$\rho \frac{D\vec{V}}{dt} = -\vec{\nabla} P - \rho \vec{g} + \nabla \cdot (\mu(T) \bar{\bar{\vec{\nabla}}} V)$$

$$\rho c_p \frac{DT}{Dt} = \nabla \cdot (\lambda(T) \vec{\nabla} T) + \sigma(T)(\vec{\nabla} U)^2$$

$$\nabla \cdot (-\sigma(T) \vec{\nabla} U) = 0. \tag{36}$$

Therefore the system is governed by only four non-dimensional numbers (A, Re, Pe and Ri).

Boundary conditions

Boundary conditions corresponding to this investigation are summarized in Fig. 1. It consists of hydrodynamic, thermal and electric boundary conditions.

Hydrodynamic boundary conditions. They consist of:

—no slip no penetration condition on the wall, i.e. $u = v = 0$;

—a velocity profile prescribed at the entrance to the system and assuming that beyond the fictive frontiers of the channel exit, the velocity does not vary. This condition can be formalized by $\partial u/\partial n = \partial v/\partial n = 0$. These boundary conditions of the homogeneous Neumann type have been naturally taken into account during the variational formulation of the problem.

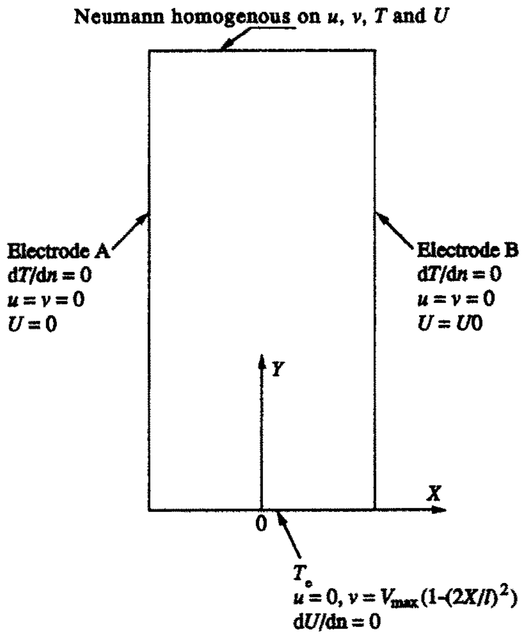

FIG. 1. Sign convention and boundary conditions in the computation domain.

Thermal boundary conditions. We have imposed:

—a uniform temperature distribution T_e at the entrance;

—a thermal insulation condition $\partial T/\partial n = 0$ at the exit from the system as well as on all channel walls; including on the electrodes. The latter boundary condition is not generally valid. In this specific case it is justified based on the fact that the electrodes are glued on an insulating material on the back. We can then neglect the conductive heat flux in the normal as well as transverse directions on the electrodes.

Electric boundary conditions. These consist of:

—a null electric potential imposed on the electrode A;

—an electric potential U_0 imposed on the electrode B;

and considering an electric insulation $\partial U/\partial n = 0$ on all other walls.

Numerical techniques

We have solved the above system of coupled equations (36) subject to the mentioned boundary conditions, by a control volume code (ULISSE) developed in the Laboratoire National d'Hydrodynamique of EDF. The code is designed to solve essentially the Navier–Stokes equations, with or without energy equation, in a two-dimensional geometry. It utilizes a fractional time step method of resolution which allows to define several computation steps and to use different numerical methods best adapted to each step [6]. A short description of the method follows.

If we approximate the time derivative by a finite difference of the first order as

$$\frac{\partial \vec{\mathbf{V}}}{\partial t} = \frac{\vec{\mathbf{V}}^{n+1} - \vec{\mathbf{V}}^{n}}{\mathrm{d}t}$$

the algorithm for computation of velocity v^{n+1} at time t^{n+1} from the velocity v^n known at time t^n can be written in three steps

$$\vec{\mathbf{V}}' - \vec{\mathbf{V}}^{n} = -\vec{\mathbf{V}}\bar{\bar{\vec{\nabla}}}\vec{\mathbf{V}} \cdot \mathrm{d}t \tag{37}$$

$$\vec{\mathbf{V}}'' - \vec{\mathbf{V}}' = \nabla \cdot \left(\frac{\mu}{\rho}\bar{\bar{\vec{\nabla}}}\vec{\mathbf{V}}''\right) \cdot \mathrm{d}t \tag{38}$$

$$\vec{\mathbf{V}}^{n+1} - \vec{V}'' = -\frac{1}{\rho}\vec{\nabla}p^{n+1}. \tag{39}$$

The sum of the three steps gives

$$\frac{\vec{\mathbf{V}}^{n+1} - \vec{\mathbf{V}}^{n}}{\mathrm{d}t} + \vec{\mathbf{V}}^{n}\bar{\bar{\vec{\nabla}}}\vec{\mathbf{V}} - \nabla \cdot \left(\frac{\mu}{\rho}\bar{\bar{\vec{\nabla}}}\vec{\mathbf{V}}''\right) + \frac{1}{\rho}\vec{\nabla}p^{n+1} = 0. \tag{40}$$

Step I, called the convection step, in which equation (37) rewritten in a different form is solved by the method of characteristics [7]. In step II, designated as the diffusion step, equation (38) is solved by the method of conjugate residues, which results in an intermediate velocity. This velocity is the real velocity less the effect of pressure field. The same technique is used to solve the diffusion term in the energy equation and in the electric equation. In step III, named the pressure step, equation (39) combined with the equation of continuity (div $\vec{\mathbf{V}}^{n+1} = 0$) results in a Laplace equation which is solved by the method of conjugate residues.

We have also developed a specific sub-program in order to take into account the potential electric equation and to solve the global system of equations. As a first case, we have solved the system of equations (36) in a rectangular geometry for the following boundary conditions

—the two vertical walls representing the electrodes are thermally insulated. No slip and no penetration conditions for the velocity and a uniform potential difference (0 on one electrode and U_0 on the other one) are imposed on the electrode walls;

—at the channel inlet, temperature and velocity profiles are uniform and parabolic, respectively, and the entrance is electrically insulated;

—at the exit natural boundary conditions are assumed, that is homogeneous Neumann boundary condition for velocity and temperature.

3. RESULTS AND DISCUSSION

Before proceeding to the treatment of non-Newtonian fluids, we have solved equation (36) for a Newtonian fluid, with thermophysical properties close to milk. This has been done to underline the basic characteristics of ohmic heating of liquids in a straight Poiseuille flow. For this purpose, temperature dependence of the thermophysical properties of the fluid are expressed by the following equations [8], which have been replaced in equation (36)

$$\lambda = -7.31\cdot 10^{-6}\cdot T^2+5.86\cdot 10^{-3}\cdot T-0.522$$
$$\mu = 10^{-3}\cdot \exp(1.842\cdot 10^{-5}\cdot T^2-0.0258\cdot T+6.318)$$
$$\rho = 1037\cdot(1-3\cdot 10^{-4}\cdot[T-273])$$
$$\sigma = 0.0483+0.012(T-298) \qquad (41)$$

and the problem is solved for a uniform entrance temperature of 293 K.

In Fig. 2 we have shown the velocity profile at the exit section of the channel for a Newtonian liquid with constant thermophysical properties (Fig. 2(a)) and also with temperature-dependent properties (Fig. 2(b)) for numerical values of $Pe = 4290$, $Re = 500$, $Ri = 1.3$ and $A = 2.5$. In both cases the flow enters from the bottom to a vertical straight channel of rectangular cross-section. The velocity and temperature profiles at the channel entrance are parabolic and uniform respectively. It should be mentioned that in case 1(a), free convection effects are absent since the liquid is a constant property fluid. In case (b), the flow acceleration close to the electrodes is due to the conjugate effects of free convection and reduced viscosity resulted from temperature dependence of the liquid. In order to evaluate the corresponding effect of ρ and μ, variations on the flow acceleration close to the walls, one has to perform a parametric study. It is evident that the variation of fluid properties (μ and ρ) contribute in similar manners in this acceleration and they are regrouped in the Richardson number.

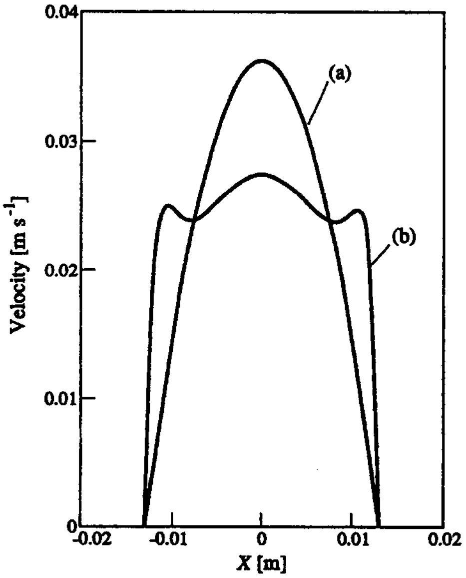

FIG. 2. Velocity profile at the exit section of the channel, (a) without the effect of natural convection, (b) with the effect of natural convection.

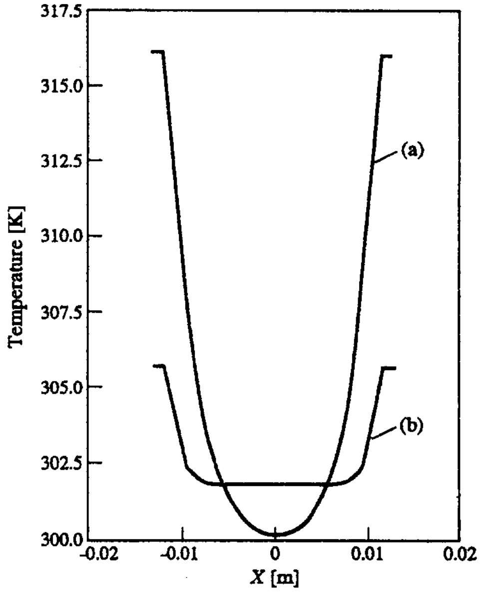

FIG. 3. Temperature profile at the exit section of the channel, (a) without the effect of natural convection, (b) with the effect of natural convection.

In the case of the temperature-dependent property liquid, the effect of natural convection due to differentially heated geometry is superimposed on the main flow. In fact, liquid particles get accelerated more in the region next to the electrodes due to the longer residence time in the channel and, therefore, higher increase in the temperature and decrease in the viscosity. The global effect of this mixed convection situation is to give a more uniform distribution of the velocity in the exit section compared with the constant property liquid. Figure 3((a) and (b)) shows the temperature profiles corresponding to the previous velocity profiles. In Fig. 3(b) the coupling between the hydrodynamic and heat transfer phenomena is quite evident. In fact, liquid acceleration close to the wall has reduced the residence time, and therefore decreased heating in this region. On the other hand, the flow slows down in the channel centre, therefore the residence time has been increased, hence more heating compared to the case of Fig. 3(a). The conjugate effect causes a more uniform distribution of temperature at the channel exit, which is desired in the continuous heating processes of the very viscous liquids.

In the following, we proceed to the treatment of non-Newtonian fluids with ohmic heating. As a model fluid we have chosen an aqueous solution of Carbopol 940 of 0.05% weight concentration. Carbopol 940 is

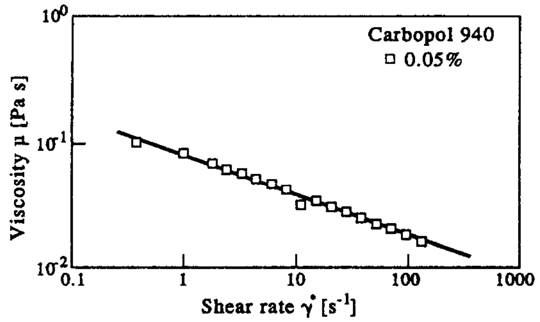

FIG. 4. Rheogram of Carbopol 940 solution at 0.05% concentration.

a Bingham plastic shear-thinning fluid whose rheological behaviour can be well expressed by the Herschel–Bulckley law [9, 10]

$$\begin{aligned} \tau - \tau_y &= \mu \cdot \dot{\gamma}^n ; \quad \tau > \tau_y \\ \dot{\gamma} &= 0 ; \quad \tau < \tau_y \end{aligned} \tag{42}$$

in which τ_y is the yield stress, which is the magnitude of the stress which must be exceeded before flow starts. In this model the apparent viscosity may be written

$$\mu = k\dot{\gamma}^{n-1} ; \quad \text{with} \quad k = a \cdot \mathrm{e}^{-bT} \tag{43}$$

with the numerical values of $a = 0.113$, $b = 0.0146$ and $n = 0.683$ [9].

A rheogram of an aqueous solution of Carbopol 940 of 0.05% concentration is shown in Fig. 4.

In order to highlight the non-Newtonian behaviour of Carbopol 940, we have defined a fictitious Newtonian fluid with an average viscosity equal to the viscosity of the non-Newtonian fluid. We designate this fictitious fluid as 'equivalent Newtonian' fluid and we define its viscosity as

$$\mu = a' \mathrm{e}^{-bT}. \tag{44}$$

Also, in order to underline the effect of temperature dependence of the non-Newtonian fluid, we have defined a second fictitious non-Newtonian fluid having the same properties as Carbopol 940 but 10-times more temperature-dependent than it. We have named it 'thermodependent' fluid, and we have defined its viscosity as

$$\mu = a'' \cdot \mathrm{e}^{-b'T} \cdot \dot{\gamma}^{n-1} ; \quad \text{with} \quad b' = 10b. \tag{45}$$

We have numerically solved equation (36) in conjunction with the above constitutive laws for the three fluids subject to the following numerical values of the nondimensional numbers: $A = 2$, $Re = 0.84$, $Pe = 1256$ and $Ri = 73.5$–84.3. The temperature rise between entrance and exit is 10°C and the mean flow velocity in the channel is 4.6 mm s^{-1}.

The numerical code solves the set of equation (36) by an iterative procedure. In this procedure, we have used the local apparent viscosity following the Herschel–Bulckley law at each iteration. Then γ is calculated and used to define $\mu = k\gamma^{n-1}$ for the next iteration. The tests of convergence are performed on the values of velocity and temperature. We have observed some convergence difficulties for the non-Newtonian fluids of high concentration (Carbopol with concentrations higher than 0.2%). In all cases we have restricted our attention to the zones far from the yield stress, as this value was not experimentally determined.

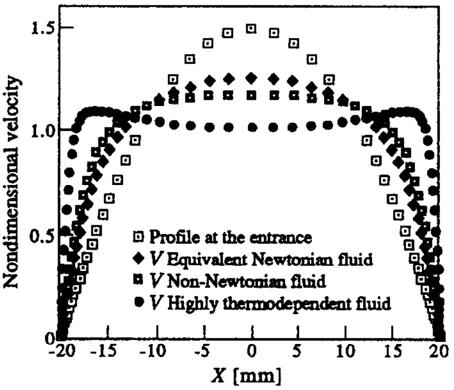

FIG. 5. Velocity profile at the entrance and at the exit sections of the channel for the non-Newtonian, equivalent Newtonian and thermodependent fluids.

On Fig. 5 we have shown the velocity profile at the channel exit for the above three fluids, all with a parabolic velocity profile at the entrance. The first observation is that the velocity profile for the non-Newtonian fluid is flatter in comparison with the 'equivalent Newtonian' fluid. The velocity profile is still flatter for the 'thermodependent' fluid. The same hierarchy of flatness prevails, in Fig. 6, for the temperature profiles at the exit, with a uniform temperature distribution at the entrance.

In order to better understand the effects of shear

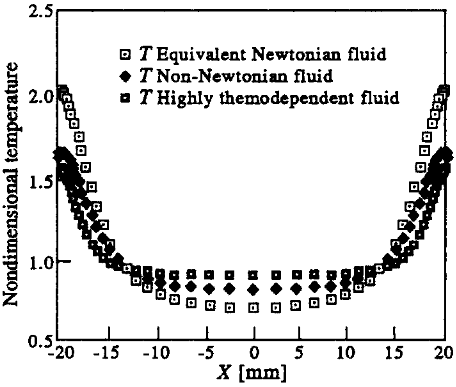

FIG. 6. Temperature profile at the entrance and at the exit sections of the channel for the non-Newtonian, equivalent Newtonian and thermodependent fluids.

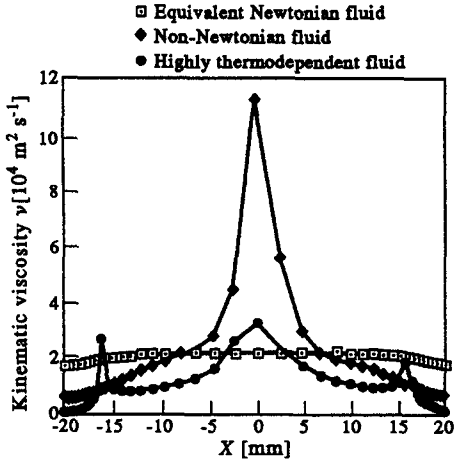

FIG. 7. Viscosity variation in the channel span for the non-Newtonian, equivalent Newtonian and thermodependent fluids.

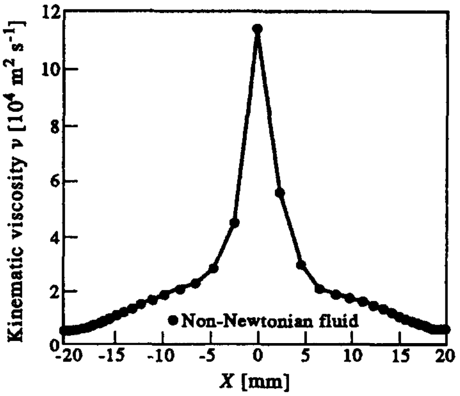

FIG. 9. Viscosity variation in the channel span for the non-Newtonian fluid.

and temperature dependence of the non-Newtonian fluids on velocity profiles, we have plotted on Fig. 7 the viscosity variation in the span at the exit plane of the channel for the three fluids. As it appears on this figure and also on Fig. 8, the viscosity of the 'equivalent Newtonian' fluid varies mildly in the channel span. On the contrary, for the case of non-Newtonian fluid (Fig. 9), under the conjugate effect of high temperature and high shear in the region close to the walls, the viscosity has diminished drastically there, giving rise to a singular point in the channel centre. On the other hand, the viscosity profile of the thermodependent fluid, as presented in Figs. 7 and 10, shows a triple peak behaviour, one in the centre and two symmetrically located close to the walls. In the region adjacent to the walls, the viscosity of the thermodependent fluid is always below that of the other two fluids. However, it is interesting to notice that it drastically increases, and eventually goes through a maximum at the spanwise position of the inflection point of the velocity profile, implying that the shear dependence dominates the temperature dependence of this fluid. The viscosity, however, relaxes beyond this point and stays always below that of the other fluids in the channel core, meaning that the temperature dependence dominates the viscosity variation in this region.

4. CONCLUSION

In this paper we have described the basic physical phenomena and derived the system of equations governing ohmic heating of ionic fluids. By using a finite volume method, we have then numerically solved the system of equations, first for a Newtonian fluid, having thermophysical properties of milk. The equations are solved in a rectangular geometry assuming Poiseuille velocity profile and uniform temperature distribution at the channel entrance. The results show acceleration of the fluid close to the elec-

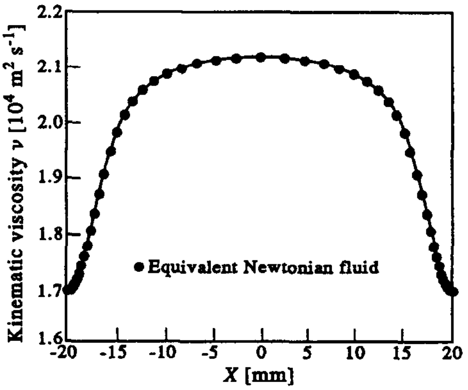

FIG. 8. Viscosity variation in the channel span for the equivalent Newtonian fluid.

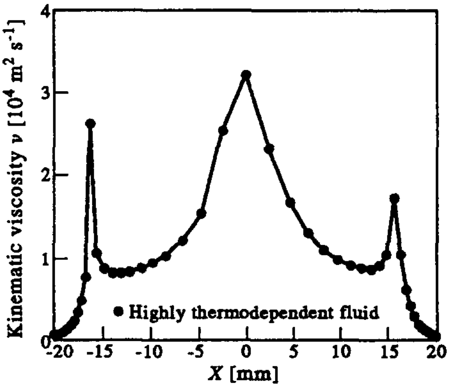

FIG. 10. Viscosity variation in the channel span for the thermodependent fluid.

trodes, emphasizing the effect of residence time and mixed convection mechanism in the system.

We have then solved the system of equations in conjunction with constitutive laws for three fluids, namely a non-Newtonian fluid following Herschel–Bulckley law, an 'equivalent Newtonian' fluid, and a highly 'thermodependent' non-Newtonian fluid. The results show clearly the equalizing effects, on the velocity and temperature profiles, of shear and temperature dependence of the non-Newtonian fluids and also the competition between shear and temperature dependence effects.

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